Paying More for a Shorter Flight? - Hidden City Ticketing

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Hidden city ticketing occurs when an indirect flight from city A to city C through connection node city B is cheaper than the direct flight from city A to city B. Then passengers traveling from A to B have an incentive to purchase the ticket from A to C but get off the plane at B. In this paper, I build a structural model to explain the cause and impact of hidden city ticketing. I collect empirical data from the Skiplagged webpage and apply global optimization algorithms to estimate the parameters of my model. I also conduct counterfactual analysis to shed some light on policy implications. I find that hidden city opportunity occurs only when airlines are applying a hub-and-spoke network structure, under which they intend to lower their flying costs compared to a fully connected network. I find that in the short run, hidden city ticketing does not necessarily decrease airlines' expected profits. Consumer welfare and total surplus always increase. In the long run, the welfare outcomes become more complicated. For some routes airlines have the incentive to switch from hub-and-spoke network to a fully connected one when there are more and more passengers informed of hidden city ticketing. During this process, firms always result in lower expected profits, while consumers and the whole society are not necessarily better off.

Keywords: hidden city ticketing, network structure, second-degree price discrimination, informed consumers.

1. Introduction

Hidden city ticketing is an interesting pricing phenomenon occurring after the deregulation of the airline industry in 1978 (Wang & Ye (2016)). It is an airline booking strategy passengers use to reduce their flying costs. Hidden city ticketing occurs when an indirect flight from city A to city C, using city B as the connection node, turns out to be cheaper than the direct flight from city A to city B. In which case passengers who wish to fly from A to B have an incentive to purchase the indirect flight ticket, pretend to fly to city C, while disembark at the connection node B, and discard the remaining segment B to C. When this happens, city B is called the "hidden city", and this behavior is then called "hidden city ticketing".

The following real world example (Figure 1) illustrates hidden city ticketing. On November 19, 2018, a direct flight operated by Delta Air Lines flying from Pittsburgh to New York city cost $218. On the same day, for the same departure and landing time, another indirect flight also operated

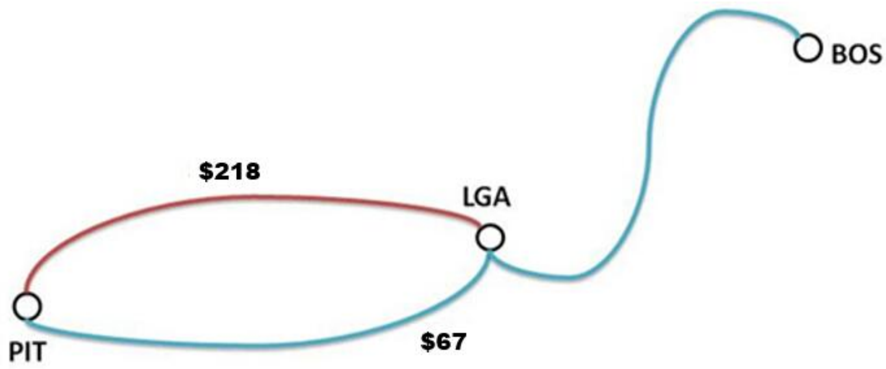


Figure 1 **An example of hidden city ticketing.**

by Delta Air Lines flying from Pittsburgh to Boston, with one stop at New York city, cost only $67.

These two flights share exactly the same first segment: they are operated by the same airline company, they departure at the same date, same airports and same time. However, the price of the indirect flight accounted for only 30% of that of the direct one. That is, you are able to fly more than 200 miles further but pay $151 less! The New York city is then called a "hidden city" in this case. It is "hidden" because literally, if we use Google Flights, Orbitz, Priceline, Kayak, or any other "normal" travel search tools to look for a flight from Pittsburgh to NYC, we will not be able to see the indirect flight above showing up in our search results.

Although technically legal, hidden city ticketing actually violates the airfare rules of most airline companies in United States. For example, according to the Contract of Carriage of United Airlines (revised by December 31, 2015):

"Fares apply for travel only between the points for which they are published. Tickets may not be purchased and used at fare(s) from an initial departure point on the Ticket which is before the Passenger's actual point of origin of travel, or to a more distant point(s) than the Passenger's actual destination being traveled even when the purchase and use of such Tickets would produce a lower fare. This practice is known as "Hidden Cities Ticketing" or "Point Beyond Ticketing" and is prohibited by UA."

Similarly, American Airlines also claim that conducting hidden city ticketing is "unethical" and doing so "is tantamount to switching price tags to obtain a lower price on goods sold at department stores". Passengers might be penalized when conducting hidden city ticketing. Airlines are able to "confiscate any unused Flight Coupons", "delete miles in the passenger's frequent flyer account", "assess the passenger for the actual value of the ticket", or even "take legal action with respect to the passenger".

Meanwhile, members of Congress have proposed several bills, including "H.R. 700, H.R. 2200, H.R. 5347 and S. 2891, H.R. 332, H.R. 384, H.R. 907 and H.R. 1074", trying to prohibit airlines from penalizing passengers for conducting hidden city ticketing (GAO (2001)). Furthermore, the European Union has passed a passenger bill of rights since around 2005, in which the European Commission has specifically ruled that "airlines must honor any part of an airline ticket" and hidden city ticketing then becomes perfectly legal. After the ruling EU find that "fares have become more fair, hidden city bargains are difficult to find, and the airlines have not suffered drastic losses due to this".

Therefore, whether hidden city ticketing should be legally prohibited or not, and what policy does the best for consumers, airline companies, and social welfare, still remain to be open questions. The fact is, although being "threatened" by airline companies, there have been more and more consumers coming to realize the existence of hidden city opportunities, and may try to exploit them to lower their flying costs. In December 2014, United Airlines and Orbitz (an airline booking platform) sued the founder of Skiplagged (a travel search tool) for his website of "helping travelers find cheap tickets through hidden city ticketing". According to CNNMoney, Orbitz eventually settled out of court one year later, and a Chicago judge threw out United's lawsuit using the excuse that the founder "did not live or do business in that city". In contrast to the willingness of United, this lawsuit brought the search of key words "hidden city ticketing" to a peak (Figure 2).

Corresponding to this higher demand, nowadays there are more travel search tools specifically designed to achieve this task (Skiplagged, Tripdelta, Fly Shortcut, AirFareIQ, ITA Matrix, etc). And finding hidden city opportunies and exploiting them become much easier today.

This paper aims at providing some plausible explanations for the cause of hidden city ticketing, and estimating its possible impact on welfare outcomes for airlines, consumers, and society as a whole. I build a structural model in which airlines can choose both prices and network structures as their strategic variables following Shy (2001), and derive several propositions based on that. Then I collect daily flights information by scraping the Skiplagged website to build my own empirical dataset. I apply global optimization algorithms to estimate the parameters of my model, and then conduct counterfactual analysis to evaluate the possible impact of hidden city ticketing on airlines' expected profits, consumers' welfare, and total surplus, based on which I could help shed some light on policy implications.

In this paper, I find that 1) hidden city ticketing only occurs when airline companies are applying a hub-and-spoke network structure; 2) under some conditions, hub-and-spoke network is more cost-saving compared to fully connected network; 3) in the short run, hidden city ticketing does not

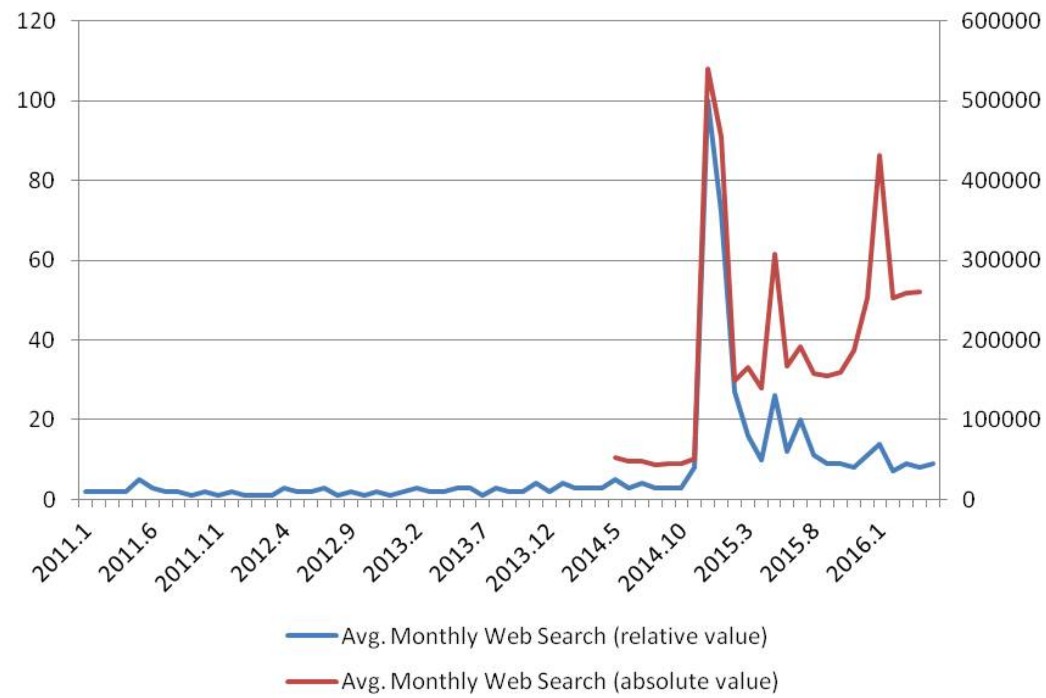


Figure 2 Average monthly web search data of hidden city ticketing. Data source for the relative value is Google Trends. Numbers represent search interest relative to the highest point on the chart for the given region and time. A value of 100 is the peak popularity for the term. Data source for the absolute value is Google AdWords, unit is number of times.

necessarily decrease airlines' expected profits, while consumers' surplus and total welfare always increase; 4) in the long run, i.e., when airlines can change their choices of prices and networks freely, the impact of hidden city ticketing differs for different routes. For some routes airlines have the incentive to switch from hub-and-spoke network to a fully connected one when there are more and more passengers informed of hidden city ticketing, during which process firms always result in lower expected revenue, while consumers and the whole society are not necessarily better off. Therefore, whether hidden city ticketing should be permitted or forbidden depends on the characteristics of different routes, and this problem cannot be solved by one simple policy.

The remainder of this paper is organized as follows: Related works are summarized in Section 2. The structural model is introduced in Section 3, together with the propositions of short run impacts derived from it. Section 4 describes the data in details. Section 5 shows the estimation strategy and the MLE results. In Section 6 I conduct counterfactual analysis to shed some light on long run impacts and policy implications. The limitation and future questions of this paper are discussed in Section 7 and Section 8 concludes.

2. Literature Review

To the best of my knowledge, this is the first paper to quantitatively study the cause and impact of hidden city ticketing on welfare outcomes using real empirical data. In fact, there are only a few papers paying attention to this phenomenon. One government report from the Government Accountability Office (GAO (2001)) conducted some correlation analysis based on their selected data, and found that the possibility of hidden city ticketing is significantly affected by the size of the markets and the degree of competition in the hub markets and the spoke markets. Another report from Hopper Research (Surry (2005)) also provides some summary statistics of this phenomenon. Based on four weeks of airfare search data from Hopper, the analyst found that 26% of domestic routes could be substituted by some cheaper options through hidden city ticketing, and the price discount could be nearly 60%. The most quantitative study is Wang & Ye (2016), which applied a network revenue management model to look at the cause and impact of hidden city ticketing. They base all their findings on simulated data rather than real world data. Therefore, their model is quite different from an economic model. They find that hidden city opportunity may arise when the price elasticity of demand on different routes differ a lot. In order to eliminate any hidden city opportunities, airlines will rise the prices of certain itineraries and hurt consumers. But even airlines optimally react, they will still suffer from a loss in revenue.

There have been a lot of literature focusing on the airline industry ever since its deregulation in 1978. A bunch of them have confirmed significant difference of price elasticity lying between tourists and business travelers. For example, Berry & Jia (2010) has estimated a price elasticity of demand for tourists as -6.55, while that for business travelers is only -0.63. Robert S & Daniel L (2001) find a large difference between price elasticity of demand for business travelers (-0.9 to -0.3) and that for leisure travelers (about -1.5). And Gerardi & Shapiro (2009) also confirm that the demand for business travelers is less price elastic than that of tourists, and through applying certain ticket restrictions, airline companies are able to distinguish between these two types. Based on these findings, researchers have further found that airlines are exploiting these differences and engaging in second-degree price discrimination through many different methods, such as advanced-purchase discounts (Dana (1998)), ticket restrictions such as Saturday-night stayover requirements (Stavins (2001); Giaume & Guillou (2004)), refundable and non-refundable tickets (Escobari & Jindapon (2014)), intertemporal price discrimination (Liu (2015); Lazarev (2013)), and even the day-of-the-week that a ticket is purchased (Puller & Taylor (2012)). This paper follows previous findings and assumes that airline companies are price discriminating between leisure travelers and business travelers, with the latter being less price sensitive and valuing time more. My model also follows Shy (2001) book about economics of network industries, assuming that airlines can choose

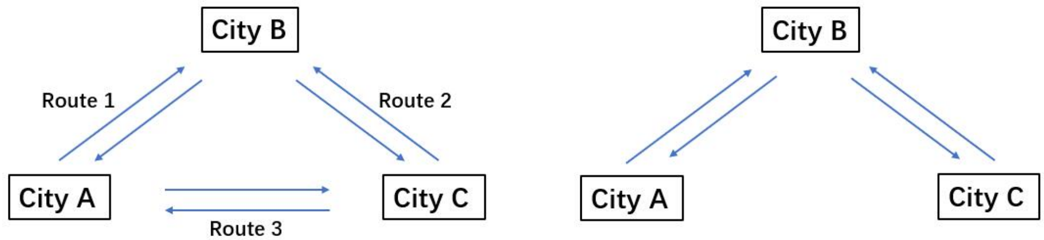


Figure 3 **Left: Fully-Connected (FC) Network. Right: Hub-and-Spoke (HS) Network.**

both airfares and network structures. Finally, I find that while informed passengers could possibly enjoy some benefits of hidden city ticketing, uninformed passengers are always bearing the costs, if any. This is similar to the finding of Varian (1980) where the author shows some "detrimental externalities" that uninformed consumers suffer from due to the behavior of informed consumers.

3. The Model

Following Shy (2001), I assume that airlines are choosing from two different network structures: fully-connected network or hub-and-spoke network (Figure 3). Under fully-connected network, passengers fly nonstop from one city to the other. While under hub-and-spoke network, everyone who wishes to fly from city A to city C needs to stop at the hub city B. To simplify my analysis below, I will apply an one-way traveling pattern instead of the two-way traveling pattern showed in Figure 3. After the 1978 Airline Deregulation Act, the absence of price and entry controls led to increased use of the hub-and-spoke structure (Shy (2001)). Responding to the increased competition and to reduce flying costs, airlines started to cut the number of direct flights and reroute the passengers through a hub city. While in recent years, especially since late 1990s, with the expansion of low-cost carriers (LCCs), passengers started to show a higher aversion toward connecting flights, and fully-connected structure becomes more popular again (Berry & Jia (2010)).

Assume that there is only one airline serving the three cities, thus the firm charges monopoly airfares. Aircrafts are further assumed to have an unlimited capacity, thus there is only one flight on each route. C_2 denote the airline's cost per mile on any route j. This simplified cost pattern is referred as ACM cost (AirCraft Movement cost) in Shy (2001), and it is widely used in airline related literature. Cost pattern can be simplified because in airline industry, large percentage of costs are fixed before flights taking off, such as capital costs (renting gates for departure and arrival, landing fees), labor costs (hiring local staff), etc. (GAO (2001)) The costs of fuel account for approximately 15% of the total operation costs (Berry & Jia (2010)), while the marginal cost of airline seats is nearly negligible (Rao (2009)).

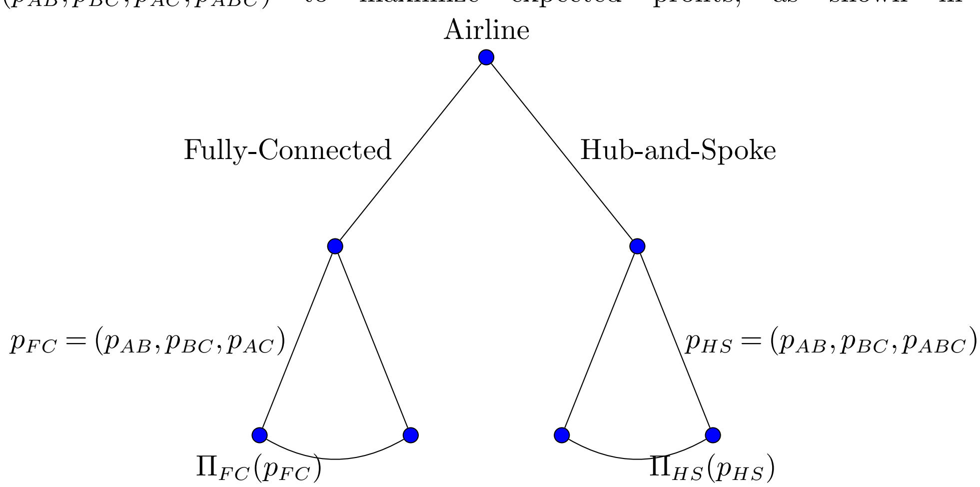
Assume that direct flight has a quality of q_h per mile and indirect flight has a quality of q_l per mile, with $0 < q_l < q_h < 1$. Each individual i has a time preference parameter of λ_i, obtaining utility

$$u_i = C_1 \cdot e^{\lambda_i} qd - p$$

from consuming a good of quality q. Under the assumption of free disposal, he/she will get 0 utility if chooses not to fly. Utility decreases when price increases. And if a passenger values time more (i.e., with a larger λ), he/she will acquire a larger utility increase when switching from an indirect flight (with quality q_l) to a direct flight (with quality q_h). Furthermore, for a longer itinerary (larger d), the utility improvement from indirect flight to direct flight is also larger. C_1 is a scaling parameter to make the utility comparable to dollar value p.

On each route j, the distribution of consumers' time preferences satisfies $\lambda_{ij} \sim N(\theta_j, \sigma_1^2)$. For passengers flying from A to B, the fraction of passengers being aware of hidden city opportunities is δ, and the fraction of uninformed passengers is $1 - \delta$. When hidden city opportunities exist (i.e., $p_{AB} > p_{ABC}$), informed passengers will pay p_{ABC} instead, while uninformed passengers will still pay p_{AB}. The amount of passengers on each route j are normalized to 1. p_j denote the airfare on route j, and d_j denote the distance of route j.

Airline chooses both network structures (fully-connected or hub-and-spoke) and prices $(p_{AB}, p_{BC}, p_{AC}, p_{ABC})$ to maximize expected profits, as shown in the figure below.



I will show later in this section that there exists an optimal choice set for the airline, and the choice set is unique. According to the assumptions above, on each route j, for each individual i,

$$u_{ij} = C_1 \cdot e^{\lambda_{ij}} qd - p, \ \lambda_{ij} \sim N(\theta_j, \sigma_1^2).$$

Therefore, on each route j, the proportion of consumers choosing to fly is equal to:

$$Pr\left[u_{ij} \geq 0\right] = Pr\left[C_1 \cdot e^{\lambda_{ij}} qd \geq p\right]$$

$$= Pr\left[\lambda_{ij} \geq ln\left(\frac{p}{C_1 \cdot qd}\right)\right]$$
$$= 1 - \Phi_{\theta_j, \sigma_1^2}\left(ln\left(\frac{p}{C_1 \cdot qd}\right)\right).$$

3.1. Fully-Connected Network

Under fully-connected network, airline's expected profits (producer surplus) are equal to the revenue it collects minus the costs:

$$\begin{aligned}
\Pi_{FC} &= \Pi_{AB} + \Pi_{BC} + \Pi_{AC} \\
&= p_{AB} \cdot \left[1 - \Phi_{\theta_{AB}, \sigma_1^2}\left(ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)\right)\right] - C_2 \cdot d_{AB} \\
&+ p_{BC} \cdot \left[1 - \Phi_{\theta_{BC}, \sigma_1^2}\left(ln\left(\frac{p_{BC}}{C_1 \cdot q_h d_{BC}}\right)\right)\right] - C_2 \cdot d_{BC} \\
&+ p_{AC} \cdot \left[1 - \Phi_{\theta_{AC}, \sigma_1^2}\left(ln\left(\frac{p_{AC}}{C_1 \cdot q_h d_{AC}}\right)\right)\right] - C_2 \cdot d_{AC}.
\end{aligned}$$

Under fully-connected network structure, the only way to fly "indirectly" from A to C is to take the two direct flights A to B and B to C together. Obviously, with $p_{AB} < p_{AB} + p_{BC}$, we can easily derive the following proposition:

Proposition 1 *Hidden city opportunity does not exist under fully-connected network structure.*

Consumer surplus is the difference between our willingness to pay and the price we actually being charged, which equals:

$$\begin{aligned}
CS_{FC} &= CS_{AB} + CS_{BC} + CS_{AC} \\
&= \int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB} - p_{AB}\right) dF(\lambda_i) \\
&+ \int_{ln\left(\frac{p_{BC}}{C_1 \cdot q_h d_{BC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{BC} - p_{BC}\right) dF(\lambda_i) \\
&+ \int_{ln\left(\frac{p_{AC}}{C_1 \cdot q_h d_{AC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AC} - p_{AC}\right) dF(\lambda_i).
\end{aligned}$$

Adding them together, our total surplus under fully-connected network is:

$$\begin{aligned}
TS_{FC} &= PS_{FC} + CS_{FC} \\
&= \int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB}\right) dF(\lambda_i) - C_2 \cdot d_{AB}
\end{aligned}$$

$$
\begin{aligned}
&+\int_{ln\left(\frac{p_{BC}}{C_1\cdot q_h d_{BC}}\right)}^{\infty}\left(C_1\cdot e^{\lambda_i}q_h d_{BC}\right)dF(\lambda_i)-C_2\cdot d_{BC}\\
&+\int_{ln\left(\frac{p_{AC}}{C_1\cdot q_h d_{AC}}\right)}^{\infty}\left(C_1\cdot e^{\lambda_i}q_h d_{AC}\right)dF(\lambda_i)-C_2\cdot d_{AC}.
\end{aligned}
$$

No transaction fee is assumed under the setting, thus the prices we pay are equal to the prices airline receives, and both cancel out.

3.2. Hub-and-Spoke Network (without Hidden City Ticketing)

Given hub-and-spoke network structure, first consider the simple case when hidden city opportunities do not exist (i.e., $p_{AB}\leq p_{ABC}$). Under this circumstances, airline's expected profits are equal to:

$$
\begin{aligned}
\Pi_{HS} &= \Pi_{AB}+\Pi_{BC}+\Pi_{ABC}\\
&= p_{AB}\cdot\left[1-\Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{AB}}{C_1\cdot q_h d_{AB}}\right)\right)\right]-C_2\cdot d_{AB}\\
&+ p_{BC}\cdot\left[1-\Phi_{\theta_{BC},\sigma_1^2}\left(ln\left(\frac{p_{BC}}{C_1\cdot q_h d_{BC}}\right)\right)\right]-C_2\cdot d_{BC}\\
&+ p_{ABC}\cdot\left[1-\Phi_{\theta_{ABC},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1\cdot q_l d_{ABC}}\right)\right)\right].
\end{aligned}
$$

Proposition 2 *If the cost associated with maintaining route AC is sufficiently large, then the hub-and-spoke network is more profitable to operate than the fully-connected network for the monopoly airline.*

Proof of Proposition 2. Compare airline's expected profits under these two different networks:

$$
\begin{aligned}
\Pi_{HS}-\Pi_{FC} &= p_{ABC}\cdot\left[1-\Phi_{\theta_{ABC},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1\cdot q_l d_{ABC}}\right)\right)\right]\\
&\quad - p_{AC}\cdot\left[1-\Phi_{\theta_{AC},\sigma_1^2}\left(ln\left(\frac{p_{AC}}{C_1\cdot q_h d_{AC}}\right)\right)\right]\\
&\quad + C_2\cdot d_{AC}.
\end{aligned}
$$

Therefore, if the last term ($C_2\cdot d_{AC}$, refers to the cost associated with maintaining route AC) is sufficiently large, hub-and-spoke network is more profitable.

This is in accordance with the findings of previous literatures. Caves et al. (1984), Brueckner et al. (1992), Brueckner & Spiller (1994), and Berry et al. (2006) all confirm the cost economies of hubbing. Under a different framework, Shy (2001) also find that hub-and-spoke network is cost-saving if the fixed cost is large enough.

Similarly, consumer surplus equals:

$$\begin{aligned}
CS_{HS} &= CS_{AB} + CS_{BC} + CS_{ABC} \\
&= \int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB} - p_{AB}\right) dF(\lambda_i) \\
&+ \int_{ln\left(\frac{p_{BC}}{C_1 \cdot q_h d_{BC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{BC} - p_{BC}\right) dF(\lambda_i) \\
&+ \int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_l d_{ABC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_l d_{ABC} - p_{ABC}\right) dF(\lambda_i).
\end{aligned}$$

Adding them together, total surplus is equal to:

$$\begin{aligned}
TS_{HS} &= PS_{HS} + CS_{HS} \\
&= \int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB}\right) dF(\lambda_i) - C_2 \cdot d_{AB} \\
&+ \int_{ln\left(\frac{p_{BC}}{C_1 \cdot q_h d_{BC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{BC}\right) dF(\lambda_i) - C_2 \cdot d_{BC} \\
&+ \int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_l d_{ABC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_l d_{ABC}\right) dF(\lambda_i).
\end{aligned}$$

3.3. Hub-and-Spoke Network (with Hidden City Ticketing)

Now consider the scenario when hidden city opportunities exist (i.e., $p_{AB} > p_{ABC}$). Firstly, is there a possibility that $p_{AB} > p_{ABC}$, in other words, are we paying more for a shorter flight sometimes? The answer is yes. To see why this might occur, recall that

$$p_{AB} = arg \max_p \; p \cdot \left[1 - \Phi_{\theta_{AB}, \sigma_1^2}\left(ln(\frac{p}{C_1 q_h d_{AB}})\right)\right],$$

$$p_{ABC} = arg \max_p \; p \cdot \left[1 - \Phi_{\theta_{ABC}, \sigma_1^2}\left(ln(\frac{p}{C_1 q_l d_{ABC}})\right)\right],$$

where $q_h > q_l$ and $d_{AB} < d_{ABC}$. For simplification, let $q_h d_{AB} = q_l d_{ABC}$ and $\sigma_1 = 1$, rewrite the problem as

$$\begin{aligned}
p &= arg \max_p \; p \cdot \left[1 - \Phi_\theta\left(ln(\frac{p}{C})\right)\right] \\
&= arg \max_p \; p \cdot \left[1 - \Phi\left(ln(\frac{p}{C}) - \theta\right)\right],
\end{aligned}$$

where C is a constant.

Let $f(p,\theta) = p \cdot \left[1 - \Phi\left(ln(\frac{p}{C}) - \theta\right)\right]$, to find out the maximizer p^*, take derivative of $f(p,\theta)$ with respect to p and make it equal 0:

$$\begin{aligned} f_p(p,\theta) &= 1 - \Phi\left(ln(\frac{p}{C}) - \theta\right) - \phi\left(ln(\frac{p}{C}) - \theta\right) \\ &= 0. \end{aligned}$$

Let $g(p,\theta) = 1 - \Phi\left(ln(\frac{p}{C}) - \theta\right) - \phi\left(ln(\frac{p}{C}) - \theta\right)$, and take derivative of $g(p,\theta)$ with respect to θ, we get

$$\begin{aligned} g_\theta(p,\theta) &= \phi\left(ln(\frac{p}{C}) - \theta\right) + \phi_\theta\left(ln(\frac{p}{C}) - \theta\right) \\ &= \phi\left(ln(\frac{p}{C}) - \theta\right)\left(1 - ln(\frac{p}{C}) + \theta\right). \end{aligned}$$

Since $\phi\left(ln(\frac{p}{C}) - \theta\right) > 0$, if $\theta > ln(\frac{p}{C}) - 1$, we would have $g_\theta(p,\theta) > 0$, hence as long as $\theta_{AB} > \theta_{ABC}$, the optimal prices would be $p_{AB} > p_{ABC}$. Therefore, under some conditions, there is a possibility that $p_{AB} > p_{ABC}$, in other words, we are paying more for a shorter flight sometimes and hidden city opportunities exist. The underlying explanation is that airlines are pricing based on demand, rather than costs.

Comparing to Section 3.2 where there is no hidden city ticketing, the difference lies in the informed passengers who wish to fly directly from A to B. Under this circumstances, airline's expected profits are equal to:

$$\begin{aligned} \Pi_{HCT} &= \Pi_{AB} + \Pi_{BC} + \Pi_{ABC} \\ &= (1-\delta) \cdot p_{AB} \cdot \left[1 - \Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)\right)\right] - C_2 \cdot d_{AB} \\ &+ p_{BC} \cdot \left[1 - \Phi_{\theta_{BC},\sigma_1^2}\left(ln\left(\frac{p_{BC}}{C_1 \cdot q_h d_{BC}}\right)\right)\right] - C_2 \cdot d_{BC} \\ &+ p_{ABC} \cdot \left[1 - \Phi_{\theta_{ABC},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1 \cdot q_l d_{ABC}}\right)\right)\right] \\ &+ \delta \cdot p_{ABC} \cdot \left[1 - \Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)\right)\right]. \end{aligned}$$

Proposition 3 *When airlines do not alter their choices of prices and network structures, hidden city ticketing does not necessarily decrease airline's expected profits.*

Proof of Proposition 3. Compare airline's expected profits with and without hidden city ticketing respectively, and compute the difference:

$$\Pi_{HCT} - \Pi_{HS} = \delta \cdot \left\{ p_{ABC} \cdot \left[1 - \Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)\right)\right]\right.$$

$$- p_{AB} \cdot \left[1 - \Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)\right)\right]\bigg\}.$$

Note that $p_{ABC} < p_{AB}$ while $\left[1 - \Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)\right)\right] > \left[1 - \Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)\right)\right]$. We find that although airline suffers from a loss when informed passengers are paying a lower price, it also obtains some gain when this lower price attracts more consumers to take the flight. How hidden city ticketing will affect airline's expected profits actually depends on the relative dominance of these two inequalities, and this conduct does not necessarily decrease airline's expected revenue.

Consumer surplus equals:

$$\begin{aligned}
CS_{HCT} &= CS_{AB} + CS_{BC} + CS_{ABC} \\
&= (1-\delta)\int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB} - p_{AB}\right) dF(\lambda_i) \\
&+ \int_{ln\left(\frac{p_{BC}}{C_1 \cdot q_h d_{BC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{BC} - p_{BC}\right) dF(\lambda_i) \\
&+ \int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_l d_{ABC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_l d_{ABC} - p_{ABC}\right) dF(\lambda_i). \\
&+ \delta\int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB} - p_{ABC}\right) dF(\lambda_i).
\end{aligned}$$

Proposition 4 *When airlines do not alter their choices of prices and network structures, consumers are always better off when hidden city ticketing is allowed.*

Proof of Proposition 4. Compute the difference of consumer surplus with and without hidden city ticketing, we have

$$\begin{aligned}
CS_{HCT} - CS_{HS} &= \delta\int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB} - p_{ABC}\right) dF(\lambda_i) \\
&- \delta\int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB} - p_{AB}\right) dF(\lambda_i) \\
&= \delta\int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)}^{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB} - p_{ABC}\right) dF(\lambda_i) \\
&+ \delta\int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} (p_{AB} - p_{ABC})\, dF(\lambda_i) > 0.
\end{aligned}$$

The increase in consumer surplus is composed of two different parts. Firstly, the existing informed passengers are now paying a lower price, which provides them extra utility gain. Secondly, some travelers who will not fly with the original price p_{AB} are now participating in this market activity, because they are informed of the lower price p_{ABC}. These new passengers also obtain utility gain, increasing the total consumer surplus.

Adding the producer surplus and consumer surplus together, we have the total surplus under the scenario of hub-and-spoke network structure with hidden city ticketing being equal to:

$$\begin{aligned} TS_{HCT} = & (1-\delta)\int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB}\right) dF(\lambda_i) - C_2 \cdot d_{AB} \\ & + \int_{ln\left(\frac{p_{BC}}{C_1 \cdot q_h d_{BC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{BC}\right) dF(\lambda_i) - C_2 \cdot d_{BC} \\ & + \int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_l d_{ABC}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_l d_{ABC}\right) dF(\lambda_i) \\ & + \delta\int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB}\right) dF(\lambda_i). \end{aligned}$$

Proposition 5 *When airlines do not alter their choices of prices and network structures, total social welfare always increase when hidden city ticketing is allowed.*

Proof of Proposition 5. Compute the difference of total surplus with and without hidden city ticketing, we have

$$\begin{aligned} TS_{HCT} - TS_{HS} = & \delta\int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB}\right) dF(\lambda_i) \\ & - \delta\int_{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)}^{\infty} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB}\right) dF(\lambda_i) \\ = & \delta\int_{ln\left(\frac{p_{ABC}}{C_1 \cdot q_h d_{AB}}\right)}^{ln\left(\frac{p_{AB}}{C_1 \cdot q_h d_{AB}}\right)} \left(C_1 \cdot e^{\lambda_i} q_h d_{AB}\right) dF(\lambda_i) > 0. \end{aligned}$$

Total surplus increase because compared to the original price p_{AB}, there are more travelers choosing to take the flight with the lower price p_{ABC}. Extra passengers obtain extra utility gain. Since I have assumed unlimited capacity for the aircrafts, the whole society benefit from this change.

With a full analysis of airline's expected profits under fully-connected network and hub-and-spoke network, with and without hidden city ticketing, we can now show that there exists an optimal choice set for the airline to maximize its producer surplus, and the solution is unique.

Proposition 6 *Under the assumptions listed at the beginning of this section, there exists an optimal choice set* $(network, p_{AB}, p_{BC}, p_{AC}, p_{ABC})$ *for the airline to maximize its expected profits, and the solution is unique.*

Proof of Proposition 6. According to our previous analysis,

$$\begin{aligned}\Pi_{FC} &= p_{AB}\cdot\left[1-\Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{AB}}{C_1\cdot q_h d_{AB}}\right)\right)\right]-C_2\cdot d_{AB}\\ &+ p_{BC}\cdot\left[1-\Phi_{\theta_{BC},\sigma_1^2}\left(ln\left(\frac{p_{BC}}{C_1\cdot q_h d_{BC}}\right)\right)\right]-C_2\cdot d_{BC}\\ &+ p_{AC}\cdot\left[1-\Phi_{\theta_{AC},\sigma_1^2}\left(ln\left(\frac{p_{AC}}{C_1\cdot q_h d_{AC}}\right)\right)\right]-C_2\cdot d_{AC}.\end{aligned}$$

$$\begin{aligned}\Pi_{HS} &= p_{AB}\cdot\left[1-\Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{AB}}{C_1\cdot q_h d_{AB}}\right)\right)\right]-C_2\cdot d_{AB}\\ &+ p_{BC}\cdot\left[1-\Phi_{\theta_{BC},\sigma_1^2}\left(ln\left(\frac{p_{BC}}{C_1\cdot q_h d_{BC}}\right)\right)\right]-C_2\cdot d_{BC}\\ &+ p_{ABC}\cdot\left[1-\Phi_{\theta_{ABC},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1\cdot q_l d_{ABC}}\right)\right)\right]\end{aligned}$$

if $p_{AB}\le p_{ABC}$, and

$$\begin{aligned}\Pi_{HS} &= (1-\delta)\cdot p_{AB}\cdot\left[1-\Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{AB}}{C_1\cdot q_h d_{AB}}\right)\right)\right]-C_2\cdot d_{AB}\\ &+ p_{BC}\cdot\left[1-\Phi_{\theta_{BC},\sigma_1^2}\left(ln\left(\frac{p_{BC}}{C_1\cdot q_h d_{BC}}\right)\right)\right]-C_2\cdot d_{BC}\\ &+ p_{ABC}\cdot\left[1-\Phi_{\theta_{ABC},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1\cdot q_l d_{ABC}}\right)\right)\right]\\ &+ \delta\cdot p_{ABC}\cdot\left[1-\Phi_{\theta_{AB},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1\cdot q_h d_{AB}}\right)\right)\right]\end{aligned}$$

if $p_{AB} > p_{ABC}$.

Note that solving for the optimal choice set $(network, p_{AB}, p_{BC}, p_{AC}, p_{ABC})$ is equivalent to firstly solving for the optimal price bundles $(p_{AB}, p_{BC}, p_{AC}, p_{ABC})$ under fully-connected network and hub-and-spoke network respectively, and further compare $\Pi_{FC}(p_{AB}, p_{BC}, p_{AC})$ and $\Pi_{HS}(p_{AB}, p_{BC}, p_{ABC})$ to determine which joint choices of network structure and prices are optimal.

Solving for the optimal price bundle $(p_{AB}, p_{BC}, p_{AC}, p_{ABC})$ to maximize expected profits Π_{FC} and Π_{HS}, when there is no hidden city ticketing, is equivalent to solving the following problem:

$$\max_p\ p\cdot\left[1-\Phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right)\right]$$

where C is a constant.

Take derivative of the objective function and make it equal 0:

$$1-\Phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right)+p\cdot\left[-\phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right)\cdot\frac{C}{p}\cdot\frac{1}{C}\right]=0$$

$$1-\Phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right)-\phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right)=0.$$

Let $x=ln(\frac{p}{C})$, $y=\Phi_{\theta,\sigma^2}(x)$, the equation above becomes a typical ODE:

$$\begin{aligned}
1-y &= \frac{dy}{dx}\\
dx &= \frac{dy}{1-y}\\
x &= -ln(1-y)+C_1\\
ln(\frac{p}{C}) &= -ln(1-y)+C_1\\
\frac{p}{C} &= e^{-ln(1-y)}\cdot e^{C_1}\\
&= \frac{e^{C_1}}{1-y}\\
p &= \frac{C\cdot e^{C_1}}{1-y}\\
p-p\cdot\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{ln(\frac{p}{C})-\theta}{\sigma\sqrt{2}}\right)\right] &= C\cdot e^{C_1}
\end{aligned}$$

where

$$\begin{aligned}
\mathrm{erf}(z) &= \frac{1}{\sqrt{\pi}}\int_{-z}^{z}e^{-t^2}dt\\
&= \frac{2}{\sqrt{\pi}}\left(z-\frac{z^3}{3}+\frac{z^5}{10}-\frac{z^7}{42}+\frac{z^9}{216}-\cdots\right)
\end{aligned}$$

by Taylor expansion.

Therefore, the optimal price bundle is solvable. To further confirm that function $f(p)=p\cdot\left[1-\Phi_{\theta,\sigma^2}\left(ln\left(\frac{p}{C\cdot q\cdot d}\right)\right)\right]$ is unimodal, I depict function $f(p)$ with parameters $\theta=0.5,\ \sigma=0.3,\ C=10,\ q=0.8,\ d=30$, as shown in Figure 4 below.

When there is hidden city ticketing, the difference lies in the optimal value of p_{ABC}. Instead of looking for a p_{ABC} that maximizes $p_{ABC}\cdot\left[1-\Phi_{\theta_{ABC},\sigma_1^2}\left(ln\left(\frac{p_{ABC}}{C_1\cdot q_l d_{ABC}}\right)\right)\right]$, we are now solving the following problem instead:

$$\max_p\ p\cdot\left[1-\Phi_{\theta_1,\sigma^2}\left(ln(\frac{p}{C_1})\right)\right]+\delta\cdot p\cdot\left[1-\Phi_{\theta_2,\sigma^2}\left(ln(\frac{p}{C_2})\right)\right]$$

where C_1 and C_2 are constants.

Take derivative of the objective function and make it equal 0:

$$1-\Phi_{\theta_1,\sigma^2}\left(ln(\frac{p}{C_1})\right)-\phi_{\theta_1,\sigma^2}\left(ln(\frac{p}{C_1})\right)+\delta\left[1-\Phi_{\theta_2,\sigma^2}\left(ln(\frac{p}{C_2})\right)-\phi_{\theta_2,\sigma^2}\left(ln(\frac{p}{C_2})\right)\right]=0.$$

Left-hand side is a function of p, $f(p)$ with $p\in[0,+\infty)$. It is continuous because it is a linear combination of probability density function and cumulative distribution function of normal distribution, which are all continuous functions.

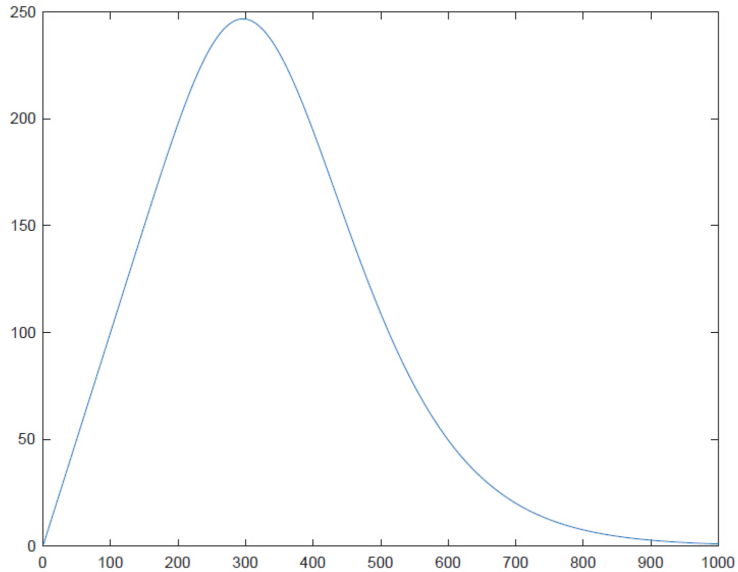


Figure 4 **Illustration of function** $f(p)$ **with parameters** $\theta = 0.5,\ \sigma = 0.3,\ C = 10,\ q = 0.8,\ d = 30.$

When $p \to 0$, $\Phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right) \to 0$, $\phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right) \in (0,1)$. Therefore, $f(p) > 0$. When $p \to +\infty$, $\Phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right) \to 1$, $\phi_{\theta,\sigma^2}\left(ln(\frac{p}{C})\right) \in (0,1)$. Therefore, $f(p) < 0$. According to Mean Value Theorem, there exists at least one p that makes $f(p) = 0$. Therefore, the solution of the model still exists.

Note that Propositions 3, 4 and 5 are derived under the assumption that airlines are not aware of hidden city ticketing, thus they do not alter their choices of prices and network structures in reaction to this booking strategy. This might be valid in the short run, while during a longer period, airlines should be able to realize the conduct of hidden city ticketing and adjust their optimal choices of prices and networks in response to the behavior. In such a scenario, obtaining a closed-form solution is challenging, and to reveal what would be the airline's optimal choice set with a changing proportion of informed passengers (changing δ) is even more difficult. Therefore, in the rest of the paper I will use numerical approach instead to solve for the optimal choices of airline with changing δ, and estimate the possible impacts of hidden city ticketing on welfare outcomes in the counterfactual analysis below.

4. Data

I have collected daily flights data by scraping the tickets information on Skiplagged webpage on February 6, 2016 with all quotes of April 6, 2016. This date was chosen because it was neither a weekend nor a holiday, and it was 60 days before the departure date, which should not be severely affected by seat sales. Information being collected include the origin, connecting (if any) and destination airports, time of departure, connection and landing, operation airlines and airfares.

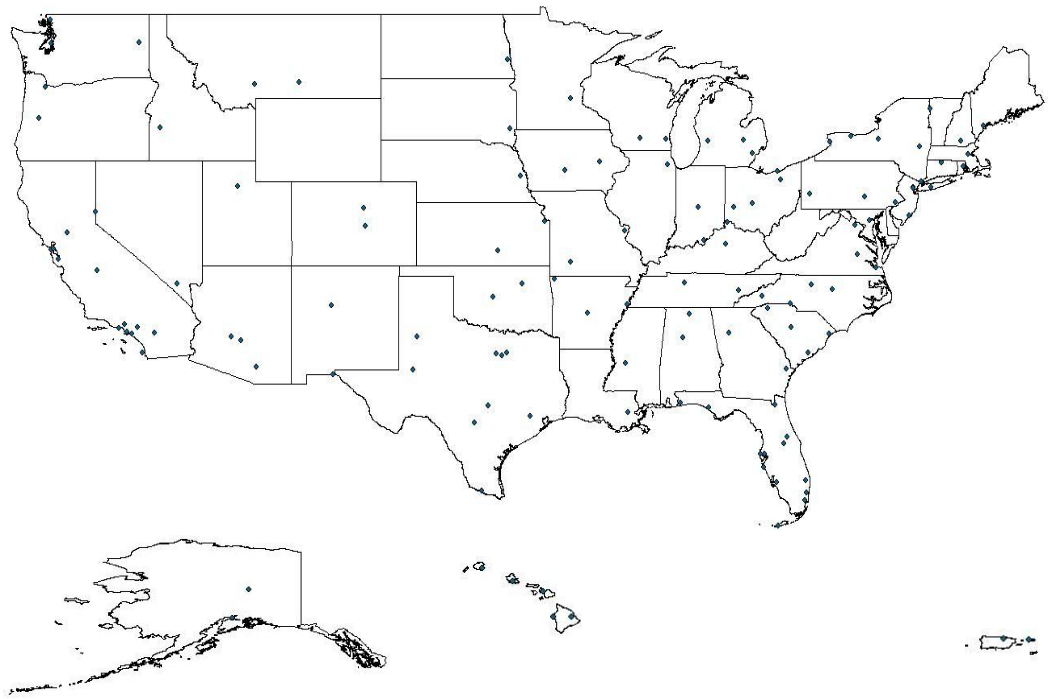


Figure 5 **Distribution of busy commercial service airports around United States.**

According to the Passenger Boardings at Commercial Service Airports of Year 2014 released in September 2015 by Federal Aviation Administration (FAA), there are more than 500 commercial service airports around United States. To reduce the computational burden of collecting data, I have restricted my sample to the 133 busy commercial service airports identified by FAA. Only focusing on those 133 airports is reasonable because those airports actually accounted for 96.34% of total passenger enplanements in 2014. The distribution of the busy commercial service airports around United States is shown in Figure 5. From the graph we can see that my data has covered airports in Alaska, Hawaii and Puerto Rico, while no airport in Wyoming has been identified as busy commercial service airport in my analysis.

Overall, my sample includes 16,142 routes (airport A to airport B) and 2,822,086 itineraries (flight from A to B with specific information of time, connection node, operation airline(s) and airfare(s)). Flights are operated by 45 different airline companies, among which 11 companies show some hidden city opportunities lying in the itineraries they operate.

To the best of my knowledge, there is no official definition of hidden city opportunity in existing literatures. GAO (2001) define that "a hidden-city ticketing opportunity exists for business travelers if the difference in airfares between the hub market and the spoke airport was $100 or more, and for leisure passengers if the difference in airfares was $50 or more". In this paper, I have constructed two intuitive definitions of hidden city opportunity myself and listed below.

Definition 1 *Hidden city opportunity exists if the cheapest non-stop ticket of that itinerary is still more expensive than some indirect flight ticket with the direct destination as a connection node.*

Definition 2 *Hidden city opportunity exists if the non-stop ticket is more expensive than some indirect flight ticket which shares exactly the same first segment of that itinerary.*

The example being illustrated at the beginning of this paper belongs to the second scenario. And the second definition is also the one being defined in Wang & Ye (2016).

According to the daily flights data I have collected, these two definitions show similar magnitude with respect to hidden city opportunities. For example, among all the itineraries, the first definition indicates a total of 366,754 (13.00%) flights and 1,095 (6.78%) routes that exhibit possible hidden city opportunities. Those amounts of Definition 2 are 394,544 (13.98%) flights and 1,316 (8.15%) routes respectively. This magnitude is slightly smaller compared to the findings in GAO (2001), in which the authors find that among the selected markets for six major U.S. passenger airlines in their data, 17% provided such opportunities. Table 1 shows the top 10 origin-destination pairs with most hidden city opportunities, which are the same under both definitions.

Table 1 **Top 10 Origin-Destination pairs with most hidden city itineraries**

Popularity	Origin	Destination	# of Itineraries	% under Def.1	% under Def.2
1	ISP	PHL	11105	3.03%	2.81%
2	SRQ	CLT	8590	2.34%	2.18%
3	CAK	CLT	5948	1.62%	1.51%
4	GRR	ORD	5733	1.56%	1.45%
5	MSN	ORD	5640	1.54%	1.43%
6	XNA	ORD	5529	1.51%	1.40%
7	COS	DEN	4000	1.09%	1.01%
8	FSD	ORD	3665	1.00%	0.93%
9	CAE	CLT	3659	1.00%	0.93%
10	ORF	CLT	3540	0.97%	0.90%

Furthermore, the maximum payment reduction would be as large as 89.57% if hidden city ticketing is allowed. Table 2 shows the top 10 origin-destination pairs with the largest price differences, which are slightly different under both definitions. These statistics help reveal the fact that hidden city ticketing might no longer be negligible nowadays and related research becomes necessary and valuable.

Table 2 **Top 10 Origin-Destination pairs with largest price differences**

Definition 1			Definition 2		
Origin	Destination	% Saving	Origin	Destination	% Saving
LGA	IAH	89.57%	LGA	IAH	89.57%
CLE	IAH	88.49%	CLE	IAH	88.49%
PHL	DTW	87.54%	PHL	DTW	87.54%
IAH	EWR	86.61%	MKE	MSP	86.65%
IAH	IAD	86.36%	IAH	EWR	86.61%
DTW	PHL	86.20%	IAH	IAD	86.36%
KOA	SFO	85.87%	DTW	PHL	86.20%
SNA	SLC	85.46%	KOA	SFO	85.87%
ICT	MSP	85.38%	SNA	SLC	85.46%
CLE	EWR	85.03%	ICT	MSP	85.38%

Recall that my primary data contains flights operated by 45 different airline companies, among which 11 companies shown some hidden city opportunities lying in the itineraries they operated. Table 3 exhibits the amounts of hidden city itineraries of these airlines unber both definitions. We can see that the three largest airlines: American Airlines, Delta Air Lines and United Airlines operated more than 99% of those itineraries. This is similar to the findings of Surry (2005), in which he found that 96% of those hidden city discounts came from American Airlines, Delta Air Lines, United Airlines and Alaska Airlines. All of them are major hub-and-spoke carriers and apply a hub-and-spoke network business model.

Table 3 **Number of hidden city itineraries of different airlines**

Airline	IATA Code	Def.1: # of Itineraries (%)	Def.2: # of Itineraries (%)
American Airlines	AA	203096 (55.38%)	210287 (53.30%)
Delta Air Lines	DL	93062 (25.38%)	106867 (27.09%)
United Airlines	UA	69587 (18.98%)	76175 (19.31%)
Alaska Airlines	AS	598 (0.16%)	666 (0.17%)
Hawaiian Airlines	HA	221 (0.06%)	221 (0.06%)
Frontier Airlines	F9	56 (0.02%)	157 (0.04%)
JetBlue Airways	B6	48 (0.01%)	106 (0.03%)
Virgin America	VX	29 (0.01%)	36 (0.01%)
Silver Airways	3M	11 (0.00%)	11 (0.00%)
Spirit Airlines	NK	8 (0.00%)	11 (0.00%)
Sun Country Airlines	SY	7 (0.00%)	7 (0.00%)

A notable exception is Southwest Airlines, where no hidden city opportunity is found in the itineraries operated by it, and whose fare rules actually do not specifically prohibit the practice of hidden city ticketing. Since Southwest Airlines is a typical operator of fully-connected network, this finding in the real data is in accordance with my previous proposition that hidden city opportunity does not exist under fully-connected network structure.

5. Estimation

To estimate the parameters of my model, firstly I retrieve all the ordered triplets (A-B-C) from my primary dataset. Then, with all the observed information of prices, distances and consumers' preferences, I choose the parameters of my model to maximize the likelihood of observed airlines' choices of network structures. In order to deal with this implicit maximum likelihood function, I have applied global optimization algorithms, more specifically, Pattern Search to solve the MLE and estimate the parameters.

5.1. Sample Build-up

To build my own sample, the first step is to retrieve ordered triplets (A-B-C) from the 133 busy commercial service airports of my primary dataset. The triplet needs to satisfy the following three conditions: 1) it must include direct flight from A to B; 2) it must include direct flight from B to C; 3) it must include either direct or one-stop indirect flight from A to C using B as the connection node.

In total, I have obtained 114,635 ordered triplets from my dataset that satisfy the conditions listed above. Based on the differences in condition 3), I divide them into three different types. Type I includes only direct flight from A to C with a subsample size of 26,198. To estimate p_{ABC} of Type I, I add observed p_{AB} and p_{BC} up manually. Type II includes only indirect flight from A to C through B with a subsample size of 61,092. To estimate p_{AC} of Type II, I use the observed p_{AB} and assume flights from A to B and A to C share the same price per mile: $p_{AC} = \dfrac{p_{AB}}{d_{AB}} \cdot d_{AC}$. Type III includes both direct flight from A to C and indirect flight from A to C through B, with a subsample size of 27,345. d represents geodesic distance computed based on the longitude and latitude of the pair-airports provided by Google Maps.

On each route j, assume that $\theta_j \sim N(\mu_j, \sigma_2^2)$. Recall that each individual i has a time preference parameter of λ_i and on each route j, the distribution of consumers' time preferences satisfies $\lambda_{ij} \sim N(\theta_j, \sigma_1^2)$. Therefore, μ_j measures the dependency of the destination city on business travelers. Previous literature have constructed several indexes to capture this characteristic. For example, Borenstein (1989) and Borenstein & Rose (1994) built a tourism index at the MSA level based on the ratio of hotel income to total personal income. Brueckner et al. (1992) and Stavins (2001) assumed that the difference in January temperature between origin and destination cities could

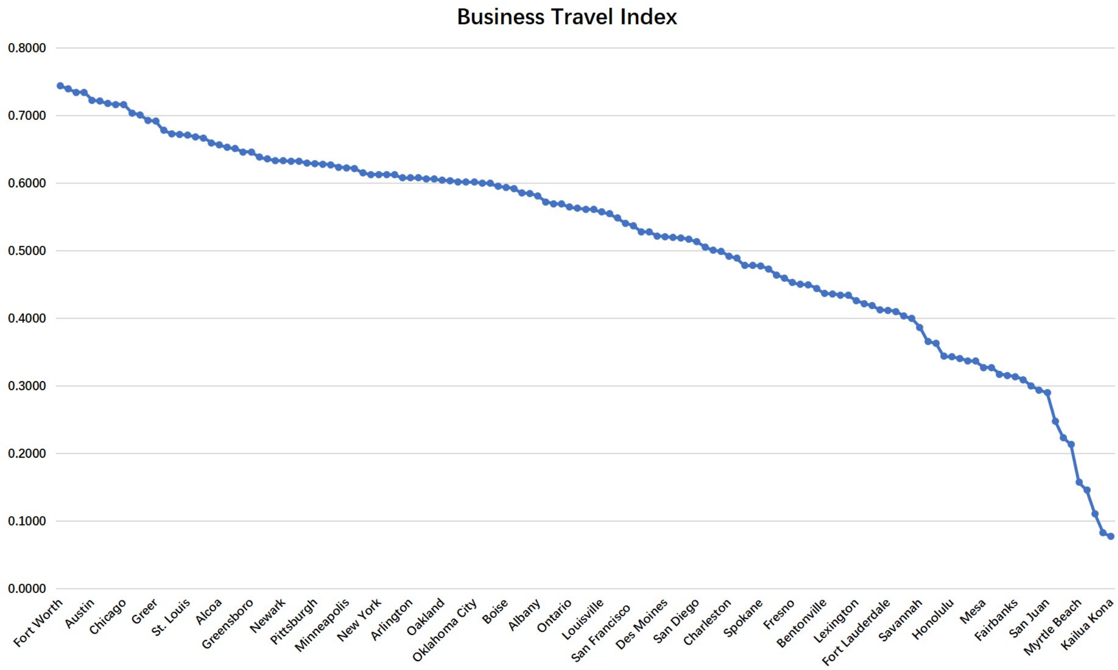


Figure 6 **Business travel index for each airport as the destination city.**

serve as a proxy for tourism. Gerardi & Shapiro (2009) segmented their data into "leisure routes" and "big-city routes" based on the ratio of accommodation earnings to total nonfarm earnings.

In this paper, I have constructed my own index based on Borenstein (2010) and data provided by TripAdvisor. Borenstein (2010) provides an index of the share of commercial airline travel to and from cities that is for business purposes, which is based on the 1995 American Travel Survey. This index was also used as one of the measures in Puller & Taylor (2012) to distinguish between "leisure" and "mixed" routes. The shortage for this index is that it only includes data for each state and metropolitan statistical area, while city level data might be a better fit corresponding to the location of an airport. To solve this problem, I have also collected data from TripAdvisor (the largest travel site in the world) for each city, and compute the average number of the reviews of hotels/lodging, vocation rentals, things to do, restaurants, and posts of forum, standardized by the city population from 2010 census. The underlying assumption is that a larger number of reviews on TripAdvisor might be an indicator of being more popular among leisure travelers, and this city-level data together with the indices constructed by Borenstein (2010) should be able to provide more complete information of the city's characteristics. After taking exponential of the opposite of the average number from TripAdvisor's review data, I compute the mean of that and the indices from Borenstein (2010) (both state-level and MSA-level) and get μ.

From Figure 6 we can see that the largest $\mu = 0.7450$ belongs to Dallas Fort Worth International Airport (DFW) in Texas, while Ellison Onizuka Kona International Airport (KOA) on the Island of Hawaii has the smallest $\mu = 0.0773$. In general, places that are more popular among tourists, such as Orlando, Puerto Rico and Hawaii, get the smaller μ(s). While places such as Dallas, Austin and Chicago that are more attractive to business travelers have larger μ(s).

5.2. Maximum Likelihood Estimation

Overall, I have a set of 7 parameters: $\zeta = (\delta, C_1, C_2, \sigma_1, \sigma_2, q_h, q_l)$, with $\delta \in [0,1]$ as my parameter of interest, and the others are nuisance parameters. Observed attributes in my dataset include the prices, distances, and time preference indices on each route: $x_i = (p_{AB,BC,AC,ABC}, d_{AB,BC,AC,ABC}, \mu_{AB,BC,AC,ABC})$. And observed decision variable is the airline's network choices: $y_i \in \{FC, HS\}$.

The maximum likelihood estimation needs to be processed in 2 steps. Firstly, I sample $\theta_{AB,BC,AC,ABC}$ from the normal distribution $\mathbb{N}|_{x_i,\sigma_2}$. Then airline makes a decision to maximize expected profits:

$$y_i = \arg \max_{y \in \{FC,HS\}} \Pi(x_i, y, \zeta).$$

The maximum likelihood estimation problem is therefore:

$$\widehat{\zeta} = \arg\max_{\zeta} \frac{1}{n} \sum_{i=1}^{n} \log\, p(y_i | x_i; \zeta),$$

with the probabilistic model as

$$Pr[y_i = y | x_i, \zeta] = \underset{\theta \sim N|_{x_i,\sigma_2}}{Pr} \left[\Pi(x_i, y, \zeta, \theta) \geq \Pi(x_i, \neg y, \zeta, \theta)\right].$$

5.3. Pattern Search

This maximum likelihood estimation is challenging because the likelihood is implicit with a random sampling in the first step, and the gradient is also difficult to evaluate with respect to ζ. Here I apply global optimization algorithms to solve this MLE problem. That is, for each ζ_t, obtain an estimation of likelihood function:

$$\begin{aligned} \log\, p(y_i|x_i;\zeta_t) &= \log \underset{\theta\sim\mathbb{N}|_{x_i,\sigma_2}}{Pr} \left[\Pi(x_i, y_i, \zeta_t, \theta) \geq \Pi(x_i, \neg y_i, \zeta_t, \theta)\right] \\ &\approx \log \left\{ \frac{1}{M} \sum_{m=1}^{M} \mathbf{1} \left[\Pi(x_i, y_i, \zeta_t, \theta_m) \geq \Pi(x_i, \neg y_i, \zeta_t, \theta_m)\right] \right\}. \end{aligned}$$

I have tried several global optimization techniques including Pattern Search, Genetic Algorithm, Simulated Annealing, etc., to get the optimal set of parameters $\zeta = (\delta, C_1, C_2, \sigma_1, \sigma_2, q_h, q_l)$ that maximizes my log likelihood function. It turns out that Pattern Search works best in this case. It costs the shortest time; It achieves the maximum log likelihood; And it obtains quite similar and robust results when I change the starting point from $\delta = 0.1, 0.5$ to 0.9.

Pattern Search algorithm fits this problem quite well because firstly, it does not require the calculation of gradients of the objective function, which are quite difficult to compute in this case. Secondly, it lends itself to constraints and boundaries. For example, it could deal with the constraint that $0 < q_l < q_h < 1$ quite well in this case.

How does the Pattern Search algorithm operate? Pattern search applies polling method (MathWorks (2018)) to find out the minimum of the objective function. Starting from an initial point, it firstly generates a pattern of points, typically plus and minus the coordinate directions, times a mesh size, and center this pattern on the current point. Then, for each point in this pattern, evaluate the objective function and compare to the evaluation of the current point. If the minimum objective in the pattern is smaller than the value at the current point, the poll is successful, and the minimum point found becomes the current point. The mesh size is then doubled in order to escape from a local minimum. If the poll is not successful, the current point is retained, and the mesh size is then halved until it falls below a threshold when the iterations stop. Multiple starting points could be used to insure that a robust minimum point has been reached regardless of the choice of the initial point.

This algorithm is simple but powerful, provides a robust and straightforward method for global optimization. It works well for the maximum likelihood function in this paper, which is derivative-free with constraints and boundaries.

5.4. Estimation Results

The estimation results from Pattern Search are shown in Table 4 below.

Table 4 Results of MLE

log likelihood	-0.3023
δ	0.0373
C_1	10.0935
C_2	0.3125
σ_1	0.2094
σ_2	0.7406
q_h	0.7010
q_l	0.1125

According to the estimation results, the informed passengers account for around 3.73% of the whole population. This proportion appeals to be trivial at first glance, but it is not surprising because those are the travelers who are not only informed of hidden city ticketing, but also exploiting those opportunities, and whose behavior in fact result in affecting the choices made by airlines. And in the counterfactual analysis section below, I will further show that even a small fraction of informed passengers will affect airline's choices of network structures and prices significantly.

In order to derive the confidence interval of my parameter of interest δ, again I apply numerical approach using bootstrap to find out the standard errors. I run the MLE for 1,000 times, and for each run, sample the entire data with replacement and construct a data set of equal size. The sample mean of the 1,000 estimates of the MLE is 0.0296, and the standard error is the sample standard deviation as 0.0093. We can see that δ is significantly different from zero at 99% confidence interval.

In Figure 7, I have plotted the log likelihood as a function of δ with all other parameters being constant at their optimal values. From the figure it is clear that $\delta = 3.73\%$ is the global maximizer.

Recall that in my theoretical model, I have assumed that there is only one airline serving the three cities, thus the firm charges monopoly airfares. Liu (2015) also made similar monopoly assumption in the paper, and corresponding to this assumption, the author refined the data and only paid attention to the routes with a single carrier operating one or two flights per day. Following this idea, I also define route AB, BC, AC, or ABC as monopoly if there is only one single carrier providing services on that route. Refining my sample of ordered triplets according to this condition results in a subsample size of 36,645, comparing to the total sample size of 114,635 before. Applying the same estimation algorithm, I have solved the MLE problem again based on the monopoly subsample, and get an estimation of δ being equal to 0.0303. The result is not significantly different from the 0.0373 we obtain above from the whole sample.

Furthermore, on September 17, 2019, I collected daily flights data again by scraping the tickets information on Skiplagged webpage with all quotes of October 1, 2019, which was neither a weekend nor a holiday, and was two weeks (rather than two months) before the departure date. This different dataset provides a slightly larger estimation of δ being equal to 0.0407, which is still not significantly different from our previous result. These two analysis provide robustness of my empirical data and my estimation methodology.

6. Counterfactual Analysis

Based on our previous analysis, given a longer horizon, airlines should be able to adjust their prices and networks in response to hidden city ticketing, in order to maximize their expected

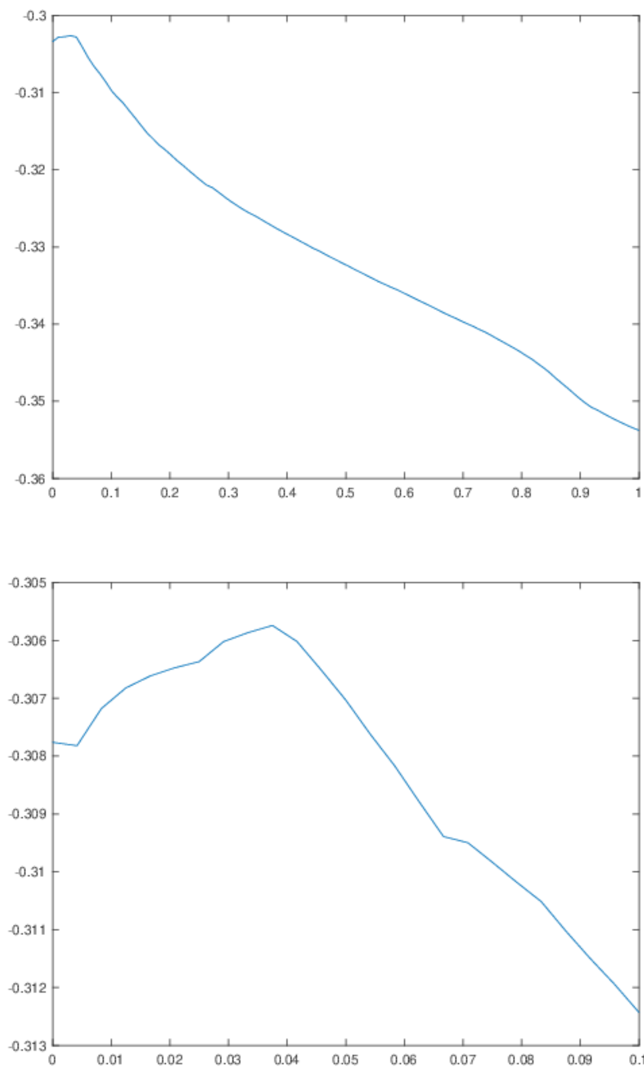


Figure 7 **Up: Plot of log likelihood when δ varies from 0 to 1. Below: Plot of log likelihood when δ varies from 0 to 0.1 (zoom in).**

profits. To reveal what would be the airline's optimal joint choices of prices and network structures when δ changes, and further estimate the possible impacts of hidden city ticketing on welfare outcomes, I have conducted several counterfactual analysis using numerical approach below. Will airline companies always suffer from revenue loss with hidden city ticketing? Will hidden city ticketing always benifit consumers and social welfare? Should government enact regulations to clearly prohibit or permit this booking ploy? My counterfactual experiments will help shed some light on those important policy implications.

Basically, assume that the proportion of informed passengers (δ) increases from 0 to 100%, and airline companies always choose optimal prices under different network structures to maximize their expected profits when δ changes. After obtaining the optimal price bundle p^* under different networks, I compute the surplus of producer, consumer, and society according to our previous analysis in Section 3. Then I plot producer surplus (blue), consumer surplus (red) and total surplus (black) under fully-connected network (dotted line) and hub-and-spoke network (solid line) respectively, when δ varies.

6.1. Fully-Connected Network Outperforms Hub-and-Spoke Network

Findings 1 *Among all the 114,635 data points (i.e., ordered triplets A-B-C), 75,995 (66.29%) have expected profits under fully-connected network being always higher than that under hub-and-spoke network, regardless of the value of δ.*

My first finding is that under major cases, fully-connected network creates higher expected profits for airlines comparing to hub-and-spoke network, regardless of the proportion of informed passengers. One example would be the ordered triplets MIA→SEA→COS (Miami International Airport to Seattle-Tacoma International Airport to Colorado Springs Airport). Figure 8 shows the surplus of producer, consumer, and society with different δ(s). The dotted lines are always horizonal because according to my model, hidden city ticketing will not affect the welfare outcomes under fully-connected network structure. It is clear that in this example, the dotted blue line is always above the solid blue one, regardless of the value of δ, which means that for airlines operating from Miami International Airport to Colorado Springs Airport, a direct flight always outperforms an indirect one through Seattle-Tacoma International Airport. This is not surprising because flying from Miami to Colorado through Seattle is counter intuitive.

When we plot the surplus in Figure 8, there is a pattern of kink, which is not uncommon and also found in other examples. Digging deep I find what happens at the kink is that airlines keep raising the price p_{ABC} in response to the increasing proportion of informed passengers, δ. Consumers benefit at first because more and more informed travelers are able to exploit the hidden city opportunities, pay lower prices and obtain extra utility. However, when the kink point is reached, p_{ABC} hits the magnitude of p_{AB} and hidden city opportunities disappear. The informed passengers can no longer obtain extra utility, while since the new p_{ABC} turns out to be higher than the original price without hidden city ticketing, those passengers flying from A to C through B also get hurt. This is similar to what is called "detrimental externalities" in Varian (1980), in which the author also found that sometimes more informed consumers would cause the price paid by uninformed consumers to increase. This finding also helps confirm the concern mentioned in GAO (2001) that allowing hidden city ticketing might lead to unintended consequences, including higher prices.

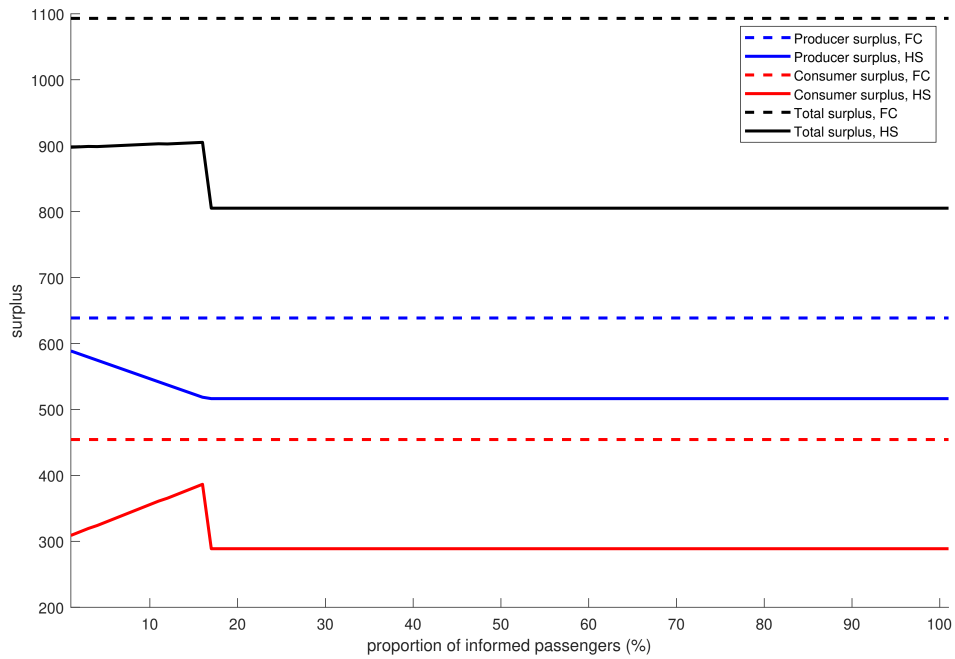


Figure 8 **Surplus for MIA→SEA→COS when δ changes.**

6.2. Hub-and-Spoke Network Outperforms Fully-Connected Network

Findings 2 *22,551 (19.67%) data points have expected profits under hub-and-spoke network being always higher than that under fully-connected network, regardless of the value of δ.*

Contradict to the previous finding, sometimes the hub-and-spoke network structure always does a better job achieving higher revenue compared to fully-connected network. One example would be the ordered triplets CID→DTW→MSN (The Eastern Iowa Airport to Detroit Metropolitan Airport to Dane County Regional Airport in Madison). Figure 9 shows the surplus of producer, consumer, and society in this case when δ varies.

We can see that the solid blue line is always above the dotted blue one, regardless of the value of δ, which means that for airlines flying from The Eastern Iowa Airport to Dane County Regional Airport in Madison, an indirect flight through Detroit Metropolitan Airport always outperforms a direct flight. This usually happens when both airports A and C are small, which is exactly what occurs when you are flying from CID to MSN. In this case, it might be costly for airlines to provide a direct flight service, especially when compared to the relatively low demand.

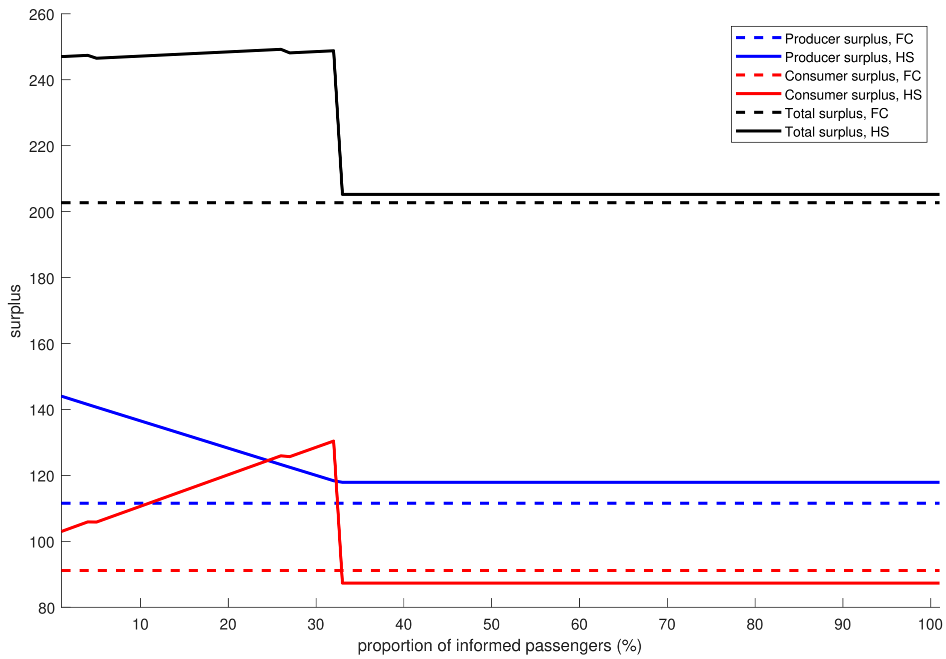


Figure 9 **Surplus for CID→DTW→MSN when δ changes.**

6.3. Switch from Hub-and-Spoke Network to Fully-Connected Network

Findings 3 *16,089 (14.03%) data points have crossings, which means that airline's expected profits are higher under hub-and-spoke network when there are less informed passengers, while fully-connected network becomes more profitable when δ gets large.*

A more interesting story lies in the cases remained: hub-and-spoke network structure is more profitable when δ is small, but becomes gradually outperformed by fully-connected network when there are more and more informed passengers. In other words, for some specific routes, airlines have the incentive to switch from one network structure to another, and δ will affect companies' network choices. This finding could be supported by what we called "dehubbing" phenomenon in recent years (Berry et al. (2006)). For example, Delta closed its Dallas-Fort Worth International Airport (DFW) hub in year 2005 and reduced the number of flights at its Cincinnati hub by 26% in the same year. And Pittsburgh was also downgraded from a hub to a "focus city" by US Airways in 2004.

One example would be the ordered triplets AUS→JFK→RDU (Austin–Bergstrom International Airport to JFK to Raleigh–Durham International Airport). Figure 10 shows the surplus of producer, consumer, and society in this case when δ varies.

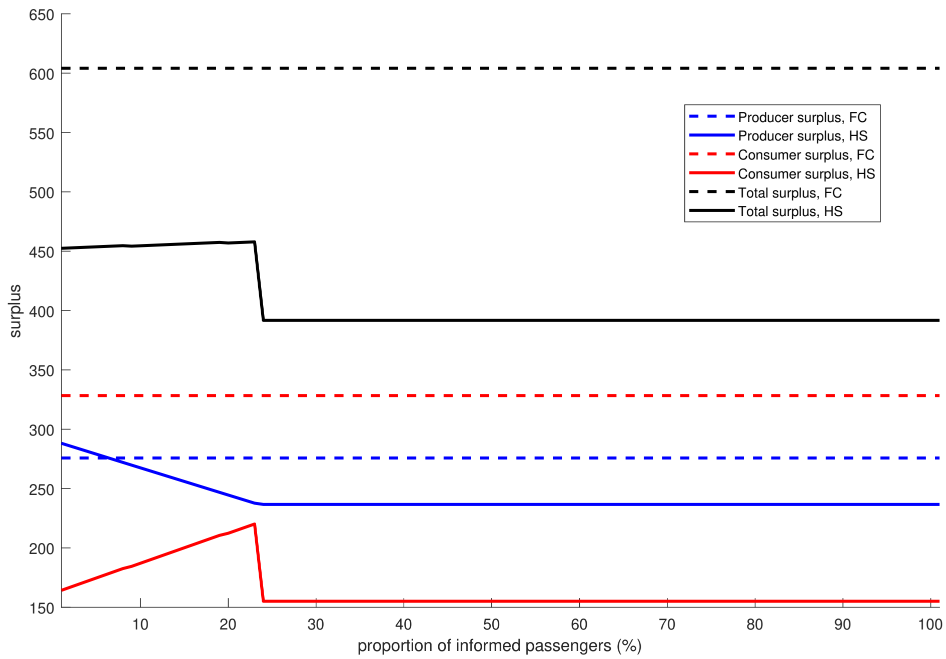


Figure 10 Surplus for AUS→JFK→RDU when δ changes.

We can see that the solid blue line crosses the dotted blue one at the point when δ is around 6%, which means that when δ is smaller than the threshhold, airline would pursue hub-and-spoke network structure. While when there are more and more informed passengers and δ crosses the threshhold, airline has the incentive to switch from the hub-and-spoke network to fully-connected network, and this decision will also affect both consumer surplus and total surplus dramatically. In this example, after the airline company making the change, both consumer surplus and total surplus increase a lot, which refer to the increase from the solid red, black lines to the dotted red, black lines respectively. But this is not always the case, and we will see more details later in this paper.

The crossing point varies for different ordered triplets. This is because different routes have different characteristics and attract different types of travelers. Some routes would be quite "sensitive" to hidden city ticketing and airlines operating on those routes would switch from hub-and-spoke network to fully-connected network when δ is relatively small. Some routes would have operating airlines changing their network choices only when the amount of informed passengers are large enough. And we have already known that sometimes airlines will never change to the fully-connected network (Findings 2), while in other cases they will stick to the fully-connected network from the very beginning (Findings 1). To have a more complete idea about the impact of different δ(s) on airlines' network choices, I can always depict the graph of surplus for every ordered triplet

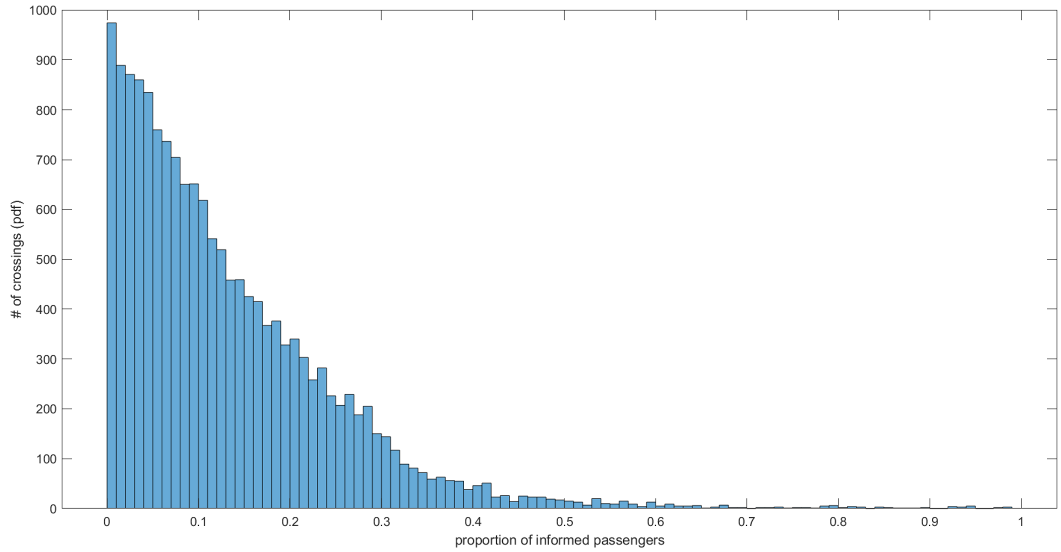


Figure 11 Distribution of crossings when δ changes (pmf)

in my data. However, it is impossible to display all those figures here (recall that I have as many as 114,635 data points in total). Therefore, in Figure 11 I have plotted the distribution of all the crossing points when δ changes.

We can see that even a small δ matters. Airlines' choices can be affected significantly even with a quite small proportion of informed passengers. For example, airlines would switch from hub-and-spoke network to fully-connected network on nearly 1,000 routes with only 1% of informed passengers, and further change their choices on another 900 routes if the proportion increases to 2%. Recall that we have obtained an estimation of $\delta = 3.73\%$ in Section 5, which appeals to be trivial at first glance, but in fact, 3% of informed passengers could affect airlines' choices of network structures on approximately 2,700 routes (out of 16,089 in my whole sample), and this amount of routes being affected would increase to around 3,600 when the proportion of informed passengers increase to 4%. To make this illustration clearer, I have also drawn the cumulative distribution function of the crossing points when δ varies from 0 to 1, and obtain my next finding from the following Figure 12.

Findings 4 *Airlines have the incentive to switch from hub-and-spoke network to fully-connected network for half of the routes when there are approximately 10% of informed passengers, and for 75% of the routes when δ is only around 19%.*

Recall that after airlines changing their choices of network structures, consumers and the whole society are not always better off. After comparing the consumer surplus and total surplus before and after the change for all those 16,089 routes, I am able to further conclude that:

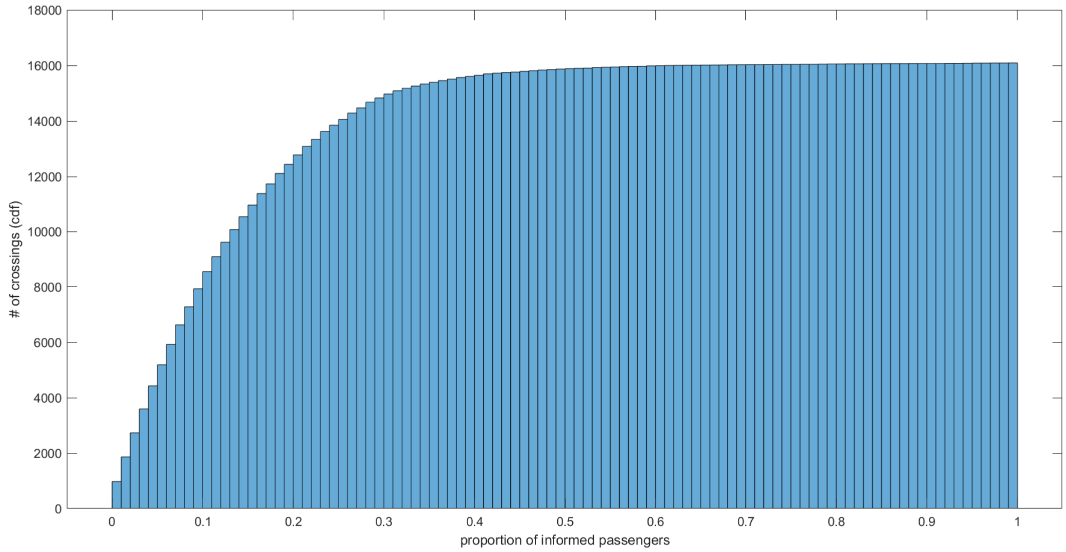


Figure 12 **Distribution of crossings when δ changes (cdf)**

Findings 5 *If airlines switch from hub-and-spoke network to fully-connected network, under 11,458 cases (71.22%) consumer surplus is going to increase, and under 11,128 (69.17%) cases total surplus is going to increase.*

In other words, unlike what could be derived from the theoretical model when airlines do not alter their choices of network structures and airfares in the short run, during a longer horizon, firms would actively react to the conduct of hidden city ticketing and change their optimal choices. This will result in different welfare outcomes for producer, consumer, and whole society compared to the propositions in Section 3. What I have found in my counterfactual analysis is that, during this process, firms always result in lower expected profits, while consumers and the whole society are not necessarily better off. Therefore, enacting a simple regulation for prohibiting or permitting the conduct of hidden city ticketing would be difficult.

7. Discussion

There are some limitations and future questions remained in this paper. Firstly, I have made a critical assumption in my model that there is only one airline serving the three cities, thus the firm charges monopoly airfares. The monopoly assumption is not uncommon in airline related literatures, but some researchers believe that after the deregulation, the US airline industry should be characterized as being hightly oligopolistic (Shy (2001)). And in GAO (2001), the authors also find that "hidden city opportunities may arise when a greater amount of competition exists for travel between spoke communities than on routes to and from hub communities, and where airfares in those markets reflect such competition". In other words, besides the factors I have raised in my propositions, competition might be another possible cause of hidden city ticketing, which does not enter my model under the monopoly assumption. And given competition, besides the cost-saving effect, another advantage of hub-and-spoke network compared to fully-connected network would be that airlines could have stronger market power in the hub, which helps them increase the entry barrier and drive up the prices for the origin-hub passengers. Since there are business travelers who favor the origin-hub route and appeal to be price-inelastic, this market power of raising prices could result in higher profits for the hubbing airlines. (Borenstein (1989), Borenstein (1991))

Another interesting question raised by Varian (1980) is that does it pay to be informed? If this is true, a better way to take this into consideration might be assuming that it is possible to become fully informed by paying a fixed cost C. Conducting hidden city tickeing is definitely costly. Passengers are "threatened" by the airline companies and need to "bear some risk" to conduct this behavior. As I have quoted from the contract of carriage in Section 1, consumers will be penalized if being caught. And hidden city ticketing might be treated as "unethical" and a breach of the contract between passengers and the airlines. There are also possible negative externalities such as causing delays of the other passengers because of waiting and double checking baggages. Furthermore, the condition of conducting hidden city ticketing is also highly restrictive. For example, if you have luggage that is not carry-on, you are not able to leave the flight earlier without picking up your bag. Normally, your checked baggage will be delivered to your final destination directly rather than to your connection city. Also, you cannot conduct hidden city ticketing for the first segment of your round-trip. Your second trip will be cancelled if you missed the connection of the first one. Besides, you might need to bear the risk that you are switched to another flight because of initial flight being cancelled or overbooked, with the same origin and destination airports, but bypass the connection city. Therefore, cost will incur to be informed might be a reasonable assumption when studying hidden city ticketing in the future, while measuring this cost would still be challenging.

Furthermore, airlines claim in the news that in reaction to the booking ploy of hidden city ticketing, they might choose to charge more on flights, stop offering some flights, and re-calibrate their no show algorithms. My analysis successfully predicts the increase in airfares of some flights. But since I have assumed that airline companies are choosing between fully-connected network and hub-and-spoke network, I do not provide an outside option for airlines to stop offering the flights for certain routes. Instead of raising the prices of flights to eliminate hidden city ticketing, another possibility is that the airlines could stop serving those defective routes. This is also another major concern in GAO (2001) that allowing hidden city ticketing might result in unintended decreasing service. Including this outside option could help make the analysis more complete, although the

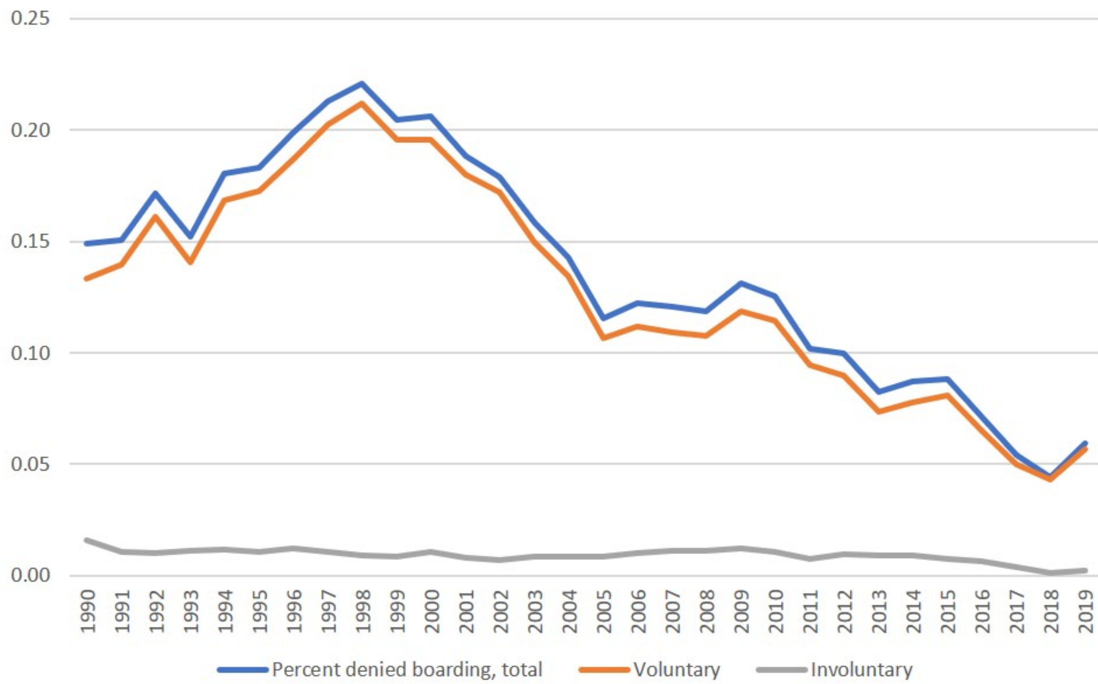


Figure 13 Percentage of Passengers Denied Boarding by the U.S. Air Carriers, 1990 to 2019.

magnitude might be difficult to evaluate without a good measure of the costs. Another concern raised by purchasing hidden city tickets is related to logistics and public-safety. When hidden city ticketing becomes more popular, airlines might need to re-calibrate their no show algorithms. They might have the incentive to oversell more, which is an act that could turn problematic and expensive if the estimates are wrong. Unfortunately, I did not find the dataset of airlines' oversales. Related statistics provided by United States Department of Transportation are numbers of passengers boarded and denied boarding by the U.S. Air Carriers. Figure 13 shows the percentage of passengers being denied boarding by the U.S. Air Carriers because of oversales, both voluntarily and involuntarily, from year 1990 to 2019.

From Figure 13 we can see a declining trend of percentage of passengers being denied boarding because of oversales in recent years, which indicates that the concern of possible oversales raised by hidden city ticketing might be subtle.

8. Conclusion

To conclude, this paper aims at analyzing the possible cause and impact of hidden city ticketing. To achieve this goal, I have constructed a structural model, collected innovative data, applied global optimization algorithm to solve the MLE, and conducted counterfactual analysis. I find that hidden city ticketing occurs only when airline companies are applying a hub-and-spoke network structure.

And airlines apply hub-and-spoke network rather than fully-connected network in order to reduce their operation costs. When airlines are not aware of hidden city ticketing, hence do not alter their choices of prices and network structures in the short run, we can derive from the theoretical model that, 1) hidden city ticketing does not necessarily decrease airline's expected profits, since the lower price also attracts more passengers to take the flight; 2) consumers are always better off when hidden city ticketing is allowed; 3) total social welfare always increase when hidden city ticketing is allowed.

During a longer horizon, firms would actively react to the conduct of hidden city ticketing and freely change their optimal choices of network structures and airfares. Under this circumstances, based on the counterfactual analysis I have conducted, I find that 1) to maximize expected profits, fully-connected network is always better than hub-and-spoke network for some routes (66.29%), while hub-and-spoke network outperforms fully-connected network for some other routes (19.67%), regardless of the proportion of informed passengers; 2) for the rest (14.03%) of the cases, airlines' expected profits are larger under hub-and-spoke network when there are less informed passengers, while fully-connected network becomes more profitable when more and more passengers starting to exploit hidden city opportunities; 3) airlines have the incentive to switch from hub-and-spoke network to fully-connected network for half of the routes when there are approximately 10% of informed passengers, and for 75% of the routes when informed passengers increase to around 19%; 4) if airlines change their network choices because of hidden city ticketing, firms are suffering from revenue loss, while consumers are not always better off (28.78% of the cases consumer surplus will decrease), and total social welfare is not always larger neither (30.83% of the cases total surplus will decrease). Therefore, enacting a simple regulation for prohibiting or permitting the conduct of hidden city ticketing would be difficult, because welfare outcome varies route by route.

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